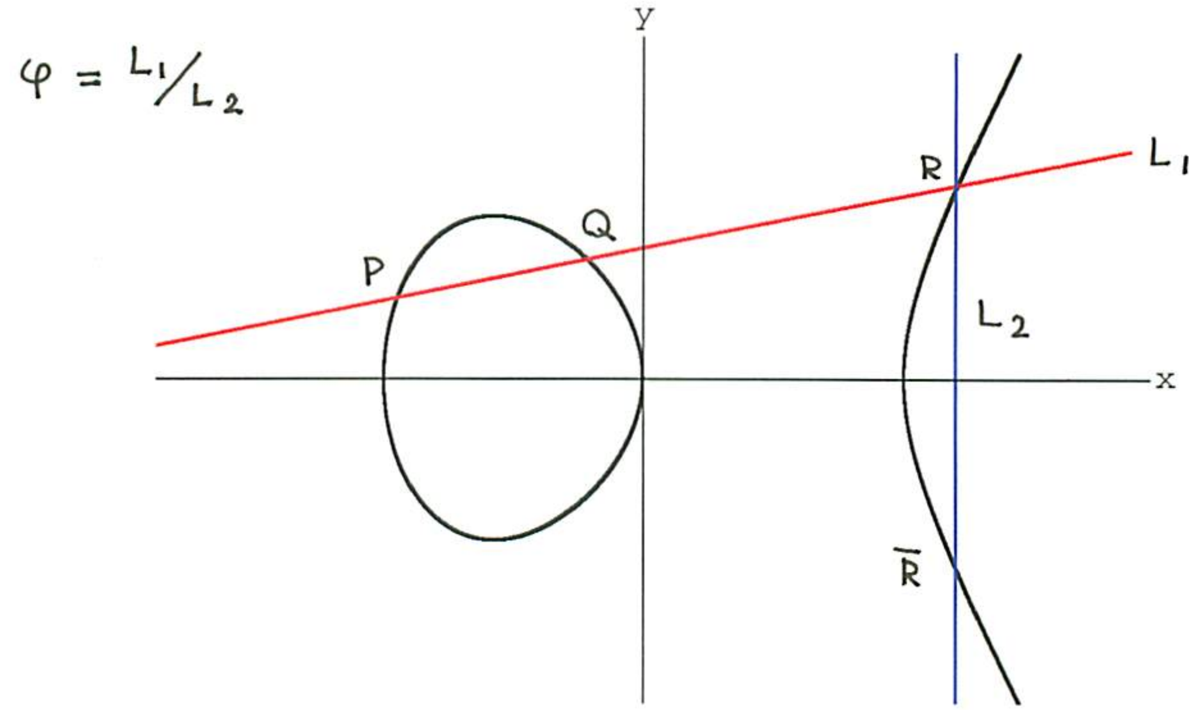
曲線のヤコビ多様体における群演算の幾何的様相
～曲線でLet's 足し算～

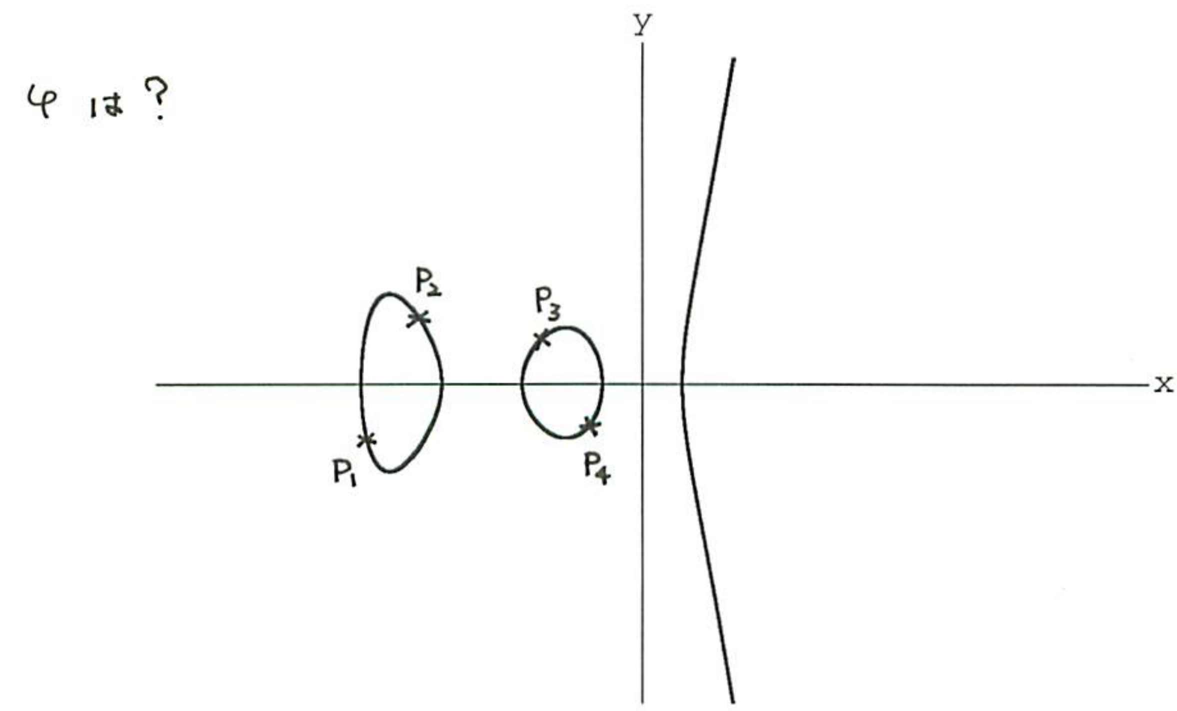
楫研究室 石川 大蔵

Example 1. ellipti curve



$$(P-\infty)+(Q-\infty)\sim\overline{R}-\infty$$

Example 2. hyperelliptic curve of g(C)=2



Theorem 1. $D_1 := \sum_{i=1}^{g} n_i P_i - g\infty,\ D_2 := \sum_{i=g+1}^{2g} n_i P_i - g\infty$ とし, 各 $P_i := (x_i, y_i)$ はすべて相異なるもので, branch point の個数は $2g-n$ 以下とする．このとき,

$$b(x) = \sum_{1 \le i_1 \lneq i_2 \lneq \cdots \lneq i_n \le 2g} y_{i_1} \cdots y_{i_n} \cdot \frac{\prod_{k \ne i_1, i_2, \cdots, i_n} (x - x_k)}{\prod_{k \ne i_1, i_2, \cdots, i_n} (x_{i_1} - x_k) \cdots (x_{i_n} - x_k)},$$

$$c(x) = \sum_{i \le i_1 \lneq i_2 \lneq \cdots \lneq i_{n-1} \le 2g} y_{i_1} \cdots y_{i_{n-1}} \cdot \frac{\prod_{l=1}^{n-1} (x_{i_l} - x)}{\prod_{k \ne i_1, i_2, \cdots, i_{n-1}} (x_{i_1} - x_k) \cdots (x_{i_{n-1}} - x_k)}.$$

特に,

$$\phi = (b(x) - y \cdot c(x))/(\prod_{i=1}^{g} L_i); \text{但し各 } L_i \text{は involution を与える直線の定義方程式.}$$

Lemma. $\mathbb{P}^3 \ni P_i := (x_i, y_i)\ (1 \le i \le N_m)$ とする． $\mathbb{C}[x,y,z,w]$ の m 次単項式を $M_1, M_2, \ldots, M_{N_m+1}$ とおく．このとき, $P_i \in H_m$ なる m 次曲面 H_m の定義方程式は,

$$F_m(P_1, P_2, \ldots, P_{N_m}) = \sum_{i=1}^{N_m+1} \det(\boldsymbol{v_1} \cdots \overbrace{-\boldsymbol{v_{N_m+1}}}^{i} \cdots \boldsymbol{v_{N_m}}) \cdot M_i \ + \ \det A \cdot M_{N_m+1}.$$

ここで, $\boldsymbol{v_i} = (M_i(P_1), \cdots, M_i(P_{N_m}))\,;\ (1 \le \forall i \le N_m),\ A = (\boldsymbol{v_1} \cdots \boldsymbol{v_{N_m}})$ で与えられる.

Theorem 2. $P_1, \ldots, P_{g+1} \in C$ を general にとる.

$$D_1 + D_2 = \sum_{i=1}^{g+1} P_i - (g+1)P_0 \sim \sum_{i=1}^{g} R_i - (g)P_0$$

を与える rational function ϕ は,
$\exists \{Q_i\}_{i=1}^{md-(g+1)},\ \exists \{R_i\}_{i=1}^{g} \subset C,\ \exists \{S_i\}_{i=1}^{N_m-md},\ \exists \{T_i\}_{i=1}^{N_m-md} \subset \mathbb{P}^3 \setminus C \quad s.t.$

$$\phi = \frac{F_m(P_1, \ldots, P_{g+1}, Q_1, \ldots, Q_{md-(g+1)}, S_1, \ldots, S_{N_m-md})}{F_m(Q_1, \ldots, Q_{md-(g+1)}, P_0, R_1, \ldots, R_g, T_1, \ldots, T_{N_m-md})}.$$

参考文献

[1] R. Hartshorne, *Algebraic Geometry*, GTM52, Springer, 1977

[2] F. Leitenberger, *About the group law for the Jacobi variety of a hyperelliptic curve*, Beiträge Algebra Geom. 46 (2005), no. 1, 125–130

[3] 佐竹一郎, 線型代数学, 裳華房, 1974