公聴会（早稲田大学　基幹理工学研究科　楫研究室　工藤陸）

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業績　　査読付き雑誌論文 2 件

- [**21**] R. Kudou, *About counterexamples for generalized Zariski cancellation problem*, Comm. Algebra, **48** (2020), 2358-2368. MR4107576
- [**22**] R. Kudou, *Zariski's cancellation problem for principal $\mathbb{G}_a$-bundles over non-$\mathbb{A}^1$-uniruled quasi-affine varieties*, Res. Math. **10** (2023), no. 1, Paper No. 2281061, 6. MR4672555

博士論文の構成

Title Zariski's cancellation problem for princpal $\mathbb{G}_a$-bundles over non-$\mathbb{A}^1$-uniruled non-affine schemes
非単線織非アフィンスキーム上の主 $\mathbb{G}_a$ 束に関するザリスキの消去問題

Chapter 1: Introduction
Chapter 2: ZCP for principal $\mathbb{G}_a$-bundles over noetherian integral scheme non-separated over k
Chapter 3: ZCP for principal $\mathbb{G}_a$-bundles over quasi-affine varieties

Chapter 1: Introduction

Problem 0.1 (Zariski's cancellation problem for an affine variety V)**.**

$$V \times_k \mathbb{A}^1 \simeq W \times_k \mathbb{A}^1 \Rightarrow V \simeq W\ ?$$

Lemma 0.2 (Danielewski's fiber product trick)**.** *Let X be a k-scheme. If two affine k-schemes V and W are isomorphic to principal $\mathbb{G}_a$-bundles over X, then $V \times_k \mathbb{A}^1 \simeq_k W \times_k \mathbb{A}^1$.*

Remark 0.3. principal $\mathbb{G}_a$-bundles over $X \leftrightarrow \mathrm{H}(X, \mathcal{O}_X)$

Definition 0.4.

- $V_g :=$ the principal $\mathbb{G}_a$-bundle over X defined by $g \in \mathrm{Z}^1(X, \mathcal{O}_X)$
- $\mathrm{P}(\overline{g}) :=$ the minimum element of the pair of numbers of poles of $g' \in \mathrm{Z}^1(X, \mathcal{O}_X)$ such that $\overline{g'} = \overline{g}$ in $\mathrm{H}^1(X, \mathcal{O}_X)$

Chapter 2: ZCP for principal $\mathbb{G}_a$-bundles over noetherian integral schemes non-separated over k

Definition 0.5. Let Y be a variety, Z a closed subvariety of Y, and $r \in \mathbb{N}$. Let $Y_0, \dots, Y_r$

be $r+1$ copies of Y. Then

$$Y_{+}rZ := Y_0 \sqcup_{Y\setminus Z} Y_1 \sqcup_{Y\setminus Z} \cdots \sqcup_{Y\setminus Z} Y_r.$$

Theorem 0.6. *Let P be a closed point of $\mathbb{A}^1_* = \mathrm{Spec}k[x, x^{-1}]$ defined by $f_1 = x - 1$. Let $X = \mathbb{A}^1_{*+}P$, $g_1 = (x+1)\cdot(x-1)^{-2}$, and $g_2 = (x-1)^{-2}$. Let V_{gi} be the principal $\mathbb{G}_a$-bundle over X defined by g_i. Then*

- $V_{g1} \not\cong V_{g2}$
- $V_{g1} \times \mathbb{A}^1 \simeq V_{g2} \times \mathbb{A}^1$
- $\mathrm{P}(\overline{g_1}) = \mathrm{P}(\overline{g_2}) = 2$

Chapter 3: ZCP for principal $\mathbb{G}_a$-bundles over quasi-affine varieties

Theorem 0.7. *Let $\mathrm{Spec}(R)$ be a non-$\mathbb{A}^1$-uniruled affine variety. Let (f_1, f_2) be an R-regular sequence, where f_1 and f_2 are prime elements such that the ideal $(f_1, f_2)_R$ is prime. Let V_g (resp. $V_{g'}$) be the principal $\mathbb{G}_a$-bundle over $D(f_1, f_2)$ that is defined by $g = v \cdot f_1^{-m} f_2^{-n}$ (resp. $g' = w \cdot f_1^{-m'} f_2^{-n'}$) with $\mathrm{P}\left(\overline{g'}\right) = (m', n')$. Then $V_g \not\cong V_{g'}$ if (1) or (2) holds.*

(1) $m' > m + n - 1$ *or* $n' > m + n - 1$

(2) $m', n' \leqq m + n - 1$ *and* $v' \notin (f_1, f_2)^{m'+n'-m-n+\delta(v)}$, *where*

$$\delta(v) = \begin{cases} 0 & \text{if } v \notin (f_1, f_2) \\ 1 & \text{if } v \in (f_1, f_2). \end{cases}$$

Corollary 0.8. *Let $\mathrm{Spec}(R)$ be a non-$\mathbb{A}^1$-uniruled affine variety. Let (f_1, f_2) be an R-regular sequence, where f_1 and f_2 are prime elements such that the ideal $(f_1, f_2)_R$ is prime. Let m, n, m', n' be integers. Then*

- $m + n \neq m' + n' \Rightarrow V_{f_1^{-m} f_2^{-n}} \not\cong V_{f_1^{-m'} f_2^{-n'}}$
- $V_{f_1^{-m} f_2^{-n}} \times \mathbb{A}^1 \simeq V_{f_1^{-m'} f_2^{-n'}} \times \mathbb{A}^1$

Corollary 0.9. *Let $\mathrm{Spec}(R)$ be a non-$\mathbb{A}^1$-uniruled affine variety. Let (f_1, f_2) be an R-regular sequence, where f_1 and f_2 are prime elements such that the ideal $(f_1, f_2)_R$ is prime. Let m, n be integers larger than 1. Let $\phi(X, Y)$ be an element of $(X, Y) \setminus ((X) \cup (Y)) \subset k[X, Y]$ satisfying $\deg_X \phi < m, \deg_Y \phi < n$. Then*

- $V_{f_1^{-m} f_2^{-n}} \not\cong V_{\phi(f_1, f_2)\cdot f_1^{-m} f_2^{-n}}$
- $V_{f_1^{-m} f_2^{-n}} \times \mathbb{A}^1 \simeq V_{\phi(f_1, f_2)\cdot f_1^{-m} f_2^{-n}} \times \mathbb{A}^1$
- $\mathrm{P}\left(\overline{f_1^{-m} f_2^{-n}}\right) = \mathrm{P}\left(\overline{\phi(f_1, f_2) \cdot f_1^{-m} f_2^{-n}}\right) = (m, n)$